Closing Wed: HW 13.3

Monday is a university holiday.

Exam 2 will be returned Tuesday

13.3 Areas between Curves

Example: Suppose

$MR(x) = -x^2 + 2x + 5$ dollars/item

$MC(x) = \frac{5}{2}x$ dollars/item

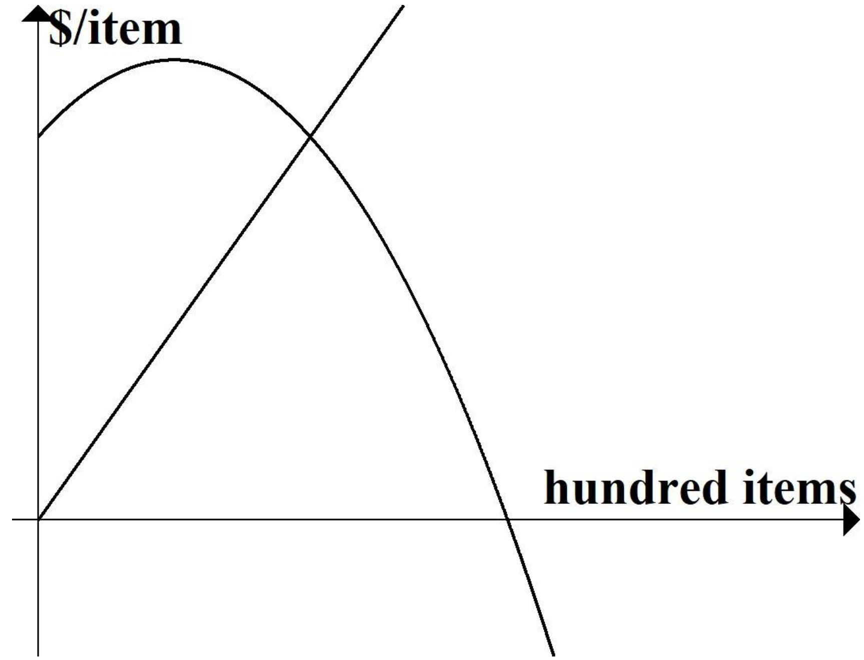
where *x* is in hundreds of items, and assume *FC* = 3 hundred dollars.

What do the following represent?

a. Area under MR from 0 to 2.

b. Area under MC from 0 to 2.

c. Area between MR & MC from 0 to 2.



Summary

$$TR(x) = \int_0^x MR(q)dq$$

$$VC(x) = \int_0^x MC(q)dq$$

$$TC(x) = \int_0^x MC(q)dq + FC$$

$$P(x) = \int_0^x MR(q)dq - \int_0^x MC(q)dq - FC$$

$$= \int_0^x MR(q) - MC(q)\ dq - FC$$

Max profit occurs at the quantity when

$$MR(q) = MC(q)$$

Specifically, when $MR(q) > MC(q)$ switches to $MR(q) < MC(q)$

And the ***value of maximum profit*** is the *net* area from 0 to this quantity minus the fixed cost.

Example:

At time t = 0 minutes, a Red and a Green balloon are next to each other at a height of 60 feet. The ***rate of ascent*** of each balloon is given by

$R'(t) = -\frac{1}{2}t^2 + 4t$ feet/min

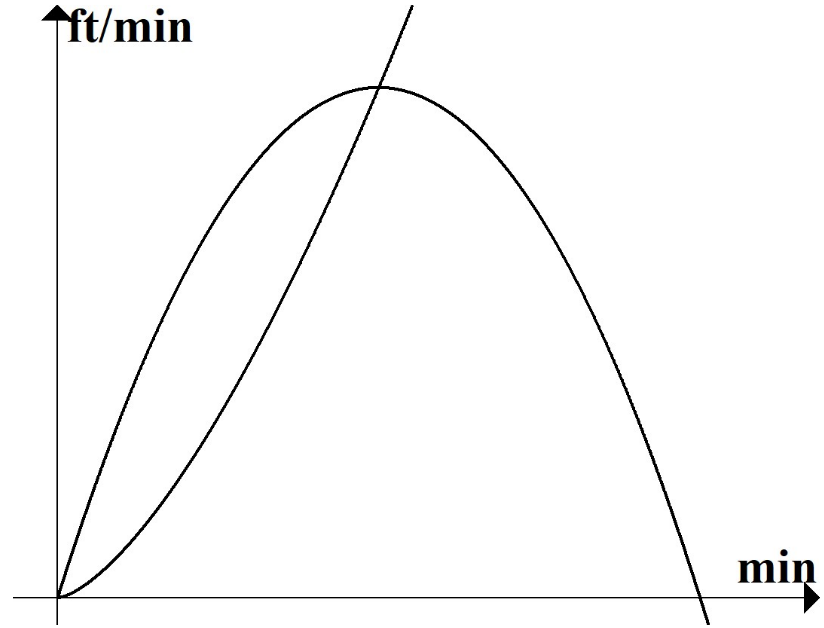
$G'(t) = t^{3/2}$ feet/min

These graphs intersect at t = 4 minutes. What do the following represent?

a. Area under $R'(t)$ from 0 to 4.

b. Area under $G'(t)$ from 0 to 4.

c. Area between from 0 to 4.



Summary

$$R(x) = \int_0^x R'(t)dt + 60$$

$$G(x) = \int_0^x G'(t)dt + 60$$

$$R(x) - G(x) = \int_0^x R'(t)dt - \int_0^x G'(t)dt$$
$$= \int_0^x R'(t) - G'(t)\, dt$$

Maximum that the red balloon is above green balloon occurs at the time when

$$R'(t) = G'(t)$$

Specifically, when $R'(t) > G'(t)$ switches to $R'(t) < G'(t)$.

And the ***value of maximum distance between*** is the *net* area from 0 to this quantity.

In general: To find area between curves

1. Draw an accurate picture.

   Find intersections and identify

   $$f(x) = \text{"top function"}$$
   $$g(x) = \text{"bottom function"}$$

2. Compute:

$$\int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b f(x) - g(x)dx$$

It gives ***change in difference between anti-derivatives*** from $x = a$ to $x = b$.

Example: Find the area of the region bounded between these curves.

$$y = x^2 - 8x + 24$$
$$y = -x^2 + 8x.$$

You do: Find the area of the region bounded by the y-axis and

$$y = 14 - 2x$$
$$y = 2 + x.$$

If *x* is in hundreds of items and

$$y = MR(x) = 14 - 2x \qquad \$/\text{item.}$$
$$y = MC(x) = 2 + x \qquad \$/\text{item.}$$

What does the area you just found represent? What additional information would you like to know?

13.4 More Integral Applications

In this section, we explore two more integral applications to business:

- Income flow
- Consumer/Supplier Surplus

Income Flow

If total income from a continuous income stream has an **annual rate** of flow given by $r(t)$, then the total income in *k* years is

$$I(k) = \int_0^k r(t)dt.$$

This formula applies if income comes in

1. "spread out" (continuous) throughout the whole year, and
2. with an annual rate $r(t)$.

Example:

1. Constant annual rate

$$r(t) = 4000 \quad \text{dollars/year}$$

What is total income in the first 5 years?

2. Linearly increasing rate

$$r(t) = 3000 + 250t \quad \text{dollars/year}$$

What is total income in the first 8 years?

3. Exponential rate (most common, *i.e.* bank account and investments)

$$r(t) = 800e^{0.05t} \quad \text{dollars/year}$$

Aside: In this model, \$800/year is the initial rate at which income is coming in and it is increasing at 5% per year (spread out throughout the year).

What is total income in the first 6 years?